Reg. No.: | | | | | | | | |

G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.

PG DEGREE END SEMESTER EXAMINATIONS - NOVEMBER 2025.
(For those admitted in June 2025 and later)

PROGRAMME AND BRANCH: M.Sc., MATHEMATICS

SEM	CATEGORY	COMPONENT	COURSE CODE	COURSE TITLE
I	PART - III	CORE ELECTIVE - 2	P25MA1E2C	ANALYTIC NUMBER THEORY

Date & Session : 12.11.2025/FN **Time : 3 hours** **Maximum: 75 Marks**

Course Outcome	Bloom's K-level	Q. No.	**SECTION – A (10 X 1 = 10 Marks)** **Answer ALL Questions.**
CO1	K1	1.	Which of the following is not true? a) $1\|n$ b) $n\|0$ c) $0\|n$ implies n=0 d) $n \nmid 0$
CO1	K2	2.	If $d\|n$ and $n \neq 0$ implies ____. a) $\|d\| \leq \|n\|$ b) $\|n\| \leq \|d\|$ c) $\|d\| = \|n\|$ d) $\|n\| < \|d\|$
CO2	K1	3.	$\varphi(10) =$ ____ where φ is Euler totient function. a) 2 b) 4 c) 6 d) 8
CO2	K2	4.	If $n \geq 1$, $\sum_{d\|n} \Lambda(d) =$____ where Λ is Mangoldt's function. a) $\log n$ b) $\log d$ c) $\varphi(n)$ d) $\varphi(d)$
CO3	K1	5.	Which one of the following is correct one($\lambda(n)$ denotes Liovillie's function)? a) $\lambda(108) = 0$ b) $\lambda(108) = 1$ c) $\lambda(108) = -1$ d) $\lambda(108) = 2$
CO3	K2	6.	A formal power series is called a _____ if all its coefficient are 0 from some point on. a) geometric series b) inverse series c) formal polynomial d) Cauchy product
CO4	K1	7.	Asymptotic value of error $E(x) = (2C-1)x + O(\sqrt{x})$ is_____. a) $\sqrt{x}$ b) $2C - 1$ c) C d) $(2C-1)x$
CO4	K2	8.	The average order of $\sigma_1(n)$ is ____. a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2 n}{12}$ c) $\frac{\pi}{6}$ d) $\frac{\pi n}{12}$
CO5	K1	9.	If $x > 0$, the Chebyshev ϑ-function is defined by the equation _____where p runs over all primes $\leq x$. a) $\vartheta(x) = \sum_{p<x} \log x$ b) $\vartheta(x) = \sum_{p \leq x} \log x$ c) $\vartheta(x) = \sum_{p<x} \log p$ d) $\vartheta(x) = \sum_{p \leq x} \log p$
CO5	K2	10.	Let p_n denote the n^{th} prime. Then $\lim_{n\to\infty} \frac{p_n}{n \log n} =$_______. a) 1 b) 0 c) n d) p

Course Outcome	Bloom's K-level	Q. No.	**SECTION – B (5 X 5 = 25 Marks)** **Answer ALL Questions choosing either (a) or (b)**
CO1	K2	11a.	Write down the statement and proof of Euclid's lemma.
			(OR)
CO1	K2	11b.	Illustrate a proof of the statement "there are infinitely many primes".

CO2	K2	12a.	If $n \geq 1$, show that $\sum_{d\|n} \varphi(d) = n$ **(OR)**
CO2	K2	12b.	Explain that mobius function is multiplicative but not completely multiplicative.
CO3	K3	13a.	Write down the proof of that if f and g are multiplicative, so is their Dirichlet product $f * g$. **(OR)**
CO3	K3	13b.	For any arithmetical function α and β, determine a proof that $\alpha \circ (\beta \circ F) = (\alpha * \beta) \circ F$
CO4	K3	14a.	For $x \geq 1$, determine that $\sum_{n \leq x} \sigma_1(n) = \frac{1}{2}\zeta(2)x^2 + O(xlog\, x)$. **(OR)**
CO4	K3	14b.	For $x > 1$, construct a proof of $\sum_{n \leq x} \varphi(n) = \frac{3}{\pi^2}x^2 + O(x \log x)$.
CO5	K4	15a.	Illustrate a proof of the Legendre's identity with its statement. **(OR)**
CO5	K4	15b.	Examine that $0 \leq \frac{\psi(x)}{x} - \frac{\vartheta(x)}{x} \leq \frac{(\log x)^2}{2\sqrt{x}\log 2}$ for $x > 0$.

Course Outcome	**Bloom's K-level**	**Q. No**	**SECTION – C (5 X 8 = 40 Marks)** **Answer ALL Questions choosing either (a) or (b)**
CO1	K4	16a.	Given any two integers a and b, examine that there is a common divisor d of a and b of the form $d = ax + by$ where x and y are integers. Moreover, inspect that every common divisor of a and b divides this d. **(OR)**
CO1	K4	16b.	Illustrate proof of the fundamental theorem of arithmetic with its statement.
CO2	K5	17a.	For $n \geq 1$, prove that $\varphi(n) = n\prod_{p\|n}\left(1 - \frac{1}{p}\right)$. **(OR)**
CO2	K5	17b.	If f is an arithmetical function with $f(1) \neq 0$,prove that there is a unique arithmetical function f^{-1}, called the Dirichlet inverse of f, such that $f * f^{-1} = f^{-1} * f = I$. Also derive the recursion formula for f^{-1}.
CO3	K5	18a.	Let f be multiplicative. Prove that f is multiplicative if and only if $f^{-1}(n) = \mu(n)f(n)$ for all $n \geq 1$. **(OR)**
CO3	K5	18b.	For every $n \geq 1$, prove that $\sum_{d\|n} \lambda(d) = \begin{cases} 1 \; if\ n\ is\ a\ square \\ 0 \quad otherwise \end{cases}$. Also, deduct that $\lambda^{-1}(n) = \|\mu(n)\|$ for all n.
CO4	K5	19a.	State and prove Euler's summation formula. **(OR)**
CO4	K5	19b.	Prove that two lattice points (a, b) and (m, n) are mutually visible if and only if $a - m$ and $b - n$ are relatively prime.
CO5	K6	20a.	Construct the proof for Abel's identity with statement. **(OR)**
CO5	K6	20b.	Propose that the following statement are logically equivalent: (i) $\lim_{x\to\infty} \frac{\pi(x)\log x}{x} = 1$ (ii) $\lim_{x\to\infty} \frac{\vartheta(x)}{x} = 1$ (iii) $\lim_{x\to\infty} \frac{\psi(x)}{x} = 1$